DPNU-95-17
UT-709
June 1995

A new method of determining $|V_{ub}|$ by the processes $\bar{B} \to \rho l \bar{\nu}$ and $\bar{B} \to K^* l \bar{l}$

A. I. Sanda

Department of Physics, Nagoya University
Chikusa-ku, Nagoya, 464-01 Japan

and

Atsushi Yamada

Department of Physics, University of Tokyo
Bunkyo-ku, Tokyo, 113 Japan

ABSTRACT

The differential decay width of the process $\bar{B} \to \rho l \bar{\nu}$ is related to that of the process $\bar{B} \to K^* l \bar{l}$ by using $SU(3)$-flavor symmetry and the heavy quark symmetry. The ratio of the Kobayashi-Maskawa matrix elements is obtained in the zero recoil limit of ρ and K^*, allowing a determination of $|V_{ub}|$.

A precise test of unitarity of the Kobayashi-Maskawa matrix [1] is essential for further investigations of the quark mass matrix and understanding the origin of CP violations. This is most conveniently performed in the B-meson systems because of the large CP-violation predicted in this system [2]. Strategies for accurate determination of Kobayashi-Maskawa matrix elements in B decay is required. An important number is one of the sides of the unitarity triangle V_{ub} [3]. A well known method is to study the leptonic spectrum at the kinematical point where the charm quark can not be produced. As there are always questions as to what extent the obtained result is independent of theoretical interpretations, it is important to get at the number in as many independent ways as possible. In this Letter, we propose a strategy to get at V_{ub}. We shall compliment the analysis of the leptonic spectrum, and propose to obtain V_{ub} from $\bar{B} \to \rho l \bar{\nu}$ and $\bar{B} \to K^* l \bar{l}$. Our analysis uses $SU(3)$-flavor and heavy quark symmetries.

Consider the zero recoil limit of the K^* and ρ mesons. Using heavy quark symmetry, it will be shown that the matrix element of the hadronic currents describing the decay $\bar{B} \to K^* l \bar{l}$ can be expressed by the same form factor appearing in the decay $\bar{B} \to \rho l \bar{\nu}$. The form factors in each process are equated the use of $SU(3)$-flavor symmetry, and the ratio of the Kobayashi-Maskawa matrix elements is obtained by the ratio of these differential decay widths. First consider the semileptonic decay $\bar{B} \to \rho l \bar{\nu}$. This process is described by the invariant amplitude

$$M = \frac{4G_F}{\sqrt{2}} V_{ub} \bar{u}_L \gamma_\mu b_L \bar{l}_L \gamma^\mu \nu. \tag{1}$$

The hadronic matrix elements required for this process are

$$\begin{aligned} < \rho(p', \epsilon) | \bar{u} \gamma_\mu b | \bar{B}(p) > &= i g^\rho \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} (p+p')^\rho (p-p')^\sigma, & (2) \\ < \rho(p', \epsilon) | \bar{u} \gamma_\mu \gamma_5 b | \bar{B}(p) > &= f^\rho {\epsilon^*}_\mu + a_+^\rho (\epsilon^* \cdot p)(p+p')_\mu + a_-^\rho (\epsilon^* \cdot p)(p-p')_\mu. & (3) \end{aligned}$$

The form factors f^ρ, $a_\pm^\rho$ and g^ρ are Lorentz invariant functions of the invariant mass squared $q^2 = (p-p')^2$ of the two leptons. The ρ meson polarization vector ϵ^* is given by

$$\epsilon_L = (\frac{\boldsymbol{p}_\rho}{m_\rho}, 0, 0, \frac{E_\rho}{m_\rho}), \quad \epsilon_\perp = (0, \epsilon_x, \epsilon_y, 0). \tag{4}$$

Hereafter we study the decay in the rest frame of the $\bar{B}$-meson, so that $p^\mu = (m_{\bar{B}}, \vec{\mathbf{0}})$,

$p'^{\mu} = (E_\rho, \vec{\boldsymbol{p}}_\rho)$ and the momentum of the ρ meson $\boldsymbol{p}_\rho = |\vec{\boldsymbol{p}}_\rho|$ is given by

$$\boldsymbol{p}_\rho = \frac{1}{2m_{\bar{B}}}[(m_{\bar{B}}^2 - m_\rho^2 - q^2)^2 - 4m_\rho^2 q^2]^{\frac{1}{2}}. \tag{5}$$

In this frame, either of $(p+p')^\rho$ and $(p-p')^\sigma$ in eq. (2) should be the spatial components $\pm\vec{\boldsymbol{p}}_\rho^i$, so the right-hand side of eq.(2) is proportional to $\boldsymbol{p}_\rho$. Furthermore, since $\epsilon^* \cdot p = \epsilon^{*0} m_{\bar{B}}$ and only the longitudinal polarization vector ϵ^L has a non-zero time component $\boldsymbol{p}_\rho/m_\rho$, the second and third terms in eq. (3) are also proportional to $\boldsymbol{p}_\rho$. Thus, the hadronic matrix elements are expanded as

$$< \rho(p', \epsilon)|\bar{u}\gamma_\mu b|\bar{B}(p) > \;=\; \mathcal{O}(\boldsymbol{p}_\rho), \tag{6}$$

$$< \rho(p', \epsilon)|\bar{u}\gamma_\mu\gamma_5 b|\bar{B}(p) > \;=\; f^\rho {\epsilon^*}_i + \mathcal{O}(\boldsymbol{p}_\rho), \tag{7}$$

in the vicinity of the zero recoil ρ meson; $\boldsymbol{p}_\rho \simeq 0$, or equivalently, the maximum q^2; $q^2 \simeq q^2_{max} = (m_{\bar{B}} - m_\rho)^2$. The q^2-distribution of the decay width is computed as

$$\frac{d\Gamma(\bar{B} \to \rho l \bar{\nu})}{dq^2} = |V_{ub}|^2 \frac{G_F}{32\pi^3 m_{\bar{B}}^2} |f^\rho|^2 q^2 \boldsymbol{p}_\rho + \mathcal{O}(\boldsymbol{p}_\rho^3). \tag{8}$$

Next we consider the flavor changing neutral decay $\bar{B} \to K^* l\bar{l}$. In the standard model, this decay takes place at the loop level via penguin and box diagrams [4]. The QCD corrected effective Hamiltonian describing this decay is

$$H \;=\; \frac{4G_F}{\sqrt{2}\pi} V_{tb}V_{ts}^* \{C_7(m_b)O_7 + C_8^{eff}(m_b)O_8 + C_9(m_b)O_9\}, \tag{9}$$

where the operators O_7, O_8 and O_9 are defined as

$$O_7 = \frac{e^2}{16\pi^2} m_b \bar{s}_L i\sigma_{\mu\nu}(q^\nu/q^2) b_R \bar{l}\gamma^\mu l, \quad O_8 = \frac{e^2}{16\pi^2} \bar{s}_L\gamma_\mu b_L \bar{l}\gamma^\mu l, \quad O_9 = \frac{e^2}{16\pi^2} \bar{s}_L\gamma_\mu b_L \bar{l}\gamma^\mu\gamma_5 l. \tag{10}$$

Here the term $m_s \bar{s}_R i\sigma_{\mu\nu}(q^\nu/q^2) b_L$ has been neglected compared to $m_b \bar{s}_L i\sigma_{\mu\nu}(q^\nu/q^2) b_R$. The QCD corrected Wilson coefficients $C_i(m_b)$ are dependent on the top quark mass. In the standard model, the vector and axial vector current operators O_8 and O_9 yield the dominant contributions to the Hamiltonian (9) and the contributions of the magnetic moment type operator O_7 is less than 10 % of O_8 and O_9. The coefficient $C_8^{eff}(m_b)$ contains the contributions from the $c\bar{c}$ continuum obtained from the electromagnetic penguin diagram and the long distance contributions due to the J/ψ and ψ' poles [5],

and therefore it is dependent on q^2. In general, the long distance contributions are significant. However, they are quite small in the regions of q^2 relevant for our analysis; $q^2 \gtrsim 0.6 m_{\bar{B}}^2$. (See e.g., figs 3(a), (b) of Lim *et. al* in Ref. [5].) The analytic expressions of the Wilson coefficients and their numerical values are given in Refs. [6]. The hadronic matrix elements of the magnetic moment type operator and the vector and axial vector currents are necessary to evaluate the Hamiltonian (9). The vector and axial vector currents are expressed in terms of the form factors as,

$$< K^*(p', \epsilon)|\bar{s}\gamma_\mu b|\bar{B}(p)> \; = \; i g^{K^*} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} (p+p')^\rho (p-p')^\sigma, \tag{11}$$

$$< K^*(p', \epsilon)|\bar{s}\gamma_\mu \gamma_5 b|\bar{B}(p)> \; = \; f^{K^*} {\epsilon^*}_\mu + a_+^{K^*} (\epsilon^* \cdot p)(p+p')_\mu + a_-^{K^*} (\epsilon^* \cdot p)(p-p')_\mu, \tag{12}$$

in the same way as in eqs. (2) and (3). Again, only the term $f^{K^*} {\epsilon^*}_i$ remains non-zero in the limit of zero recoil K^*. As for the magnetic moment type operator, since $q^\nu = (m_{\bar{B}} - m_{K^*}, \vec{\mathbf{0}}) + \mathcal{O}(\boldsymbol{p}_{K^*})$, only the components $\bar{s} i \sigma_{0i} b$ and $\bar{s} i \sigma_{0i} \gamma_5 b$ are relevant in the same limit. The hadronic matrix elements of these operators can be related to those of the vector and axial vector currents (11) and (12) by the static heavy quark approximation. In this approximation, the b quark in the $\bar{B}$ meson stays on-shell throughout the reaction, and we can set the equation of motion for b quark, $\gamma_0 b = b$, which leads to the relations [7]

$$< K^*(p', \epsilon)|\bar{s} i \sigma_{0i} b|\bar{B}(p)> \; = \; < K^*(p', \epsilon)|\bar{s}\gamma_i b|\bar{B}(p)>, \tag{13}$$

$$< K^*(p', \epsilon)|\bar{s} i \sigma_{0i} \gamma_5 b|\bar{B}(p)> \; = \; - < K^*(p', \epsilon)|\bar{s}\gamma_i \gamma_5 b|\bar{B}(p)> . \tag{14}$$

The right-hand sides of eqs. (13) and (14) can be expressed in terms of the same form factors which appeared in eqs (11) and (12), and only the term $f^{K^*} {\epsilon^*}_i$ remains non-zero in the zero recoil limit. Thus, the hadronic matrix elements required for $\bar{B} \to K^* l \bar{l}$ are described only in terms of the form factor f^{K^*} in the vicinity $\boldsymbol{p}_{K^*} \simeq 0$. The q^2-distribution of the decay width is given by

$$\frac{d\Gamma(\bar{B} \to K^* l \bar{l})}{dq^2} \; = \; |V_{tb} V_{ts}^*|^2 \frac{G_F}{32\pi^3 m_{\bar{B}}^2} |f^{K^*}|^2 (\frac{\alpha_{QED}}{4\pi})^2 2 (C_V^2 + C_A^2) q^2 \boldsymbol{p}_{K^*} + \mathcal{O}(\boldsymbol{p}_{K^*}^3), \tag{15}$$

$$C_V \; = \; -C_8^{eff}(m_b)_{q^2 = q^2_{max}} + m_b \frac{(m_{\bar{B}} - m_{K^*})}{q^2_{max}} C_7(m_b), \quad C_A = -C_9(m_b).$$

Now we extract the V_{ub} from the q^2-distributions (8) and (15). Applying the $SU(3)$-flavor symmetry to the ρ and K^* mesons. The question arises as to where we expect

the form factors f^ρ and f^{K^*} to be nearly equal. This can be settled experimentally by studying the q^2 distribution near $\boldsymbol{p}_{\rho,K^*} \to 0$ limit. For now, the best guess is that the $SU(3)$-flavor symmetry holds when u and s quarks in respective b decays to be at rest in the B meson rest frame,

$$f^\rho(q^{2\,\bar{B}\to\rho}_{max}) = f^{K^*}(q^{2\,\bar{B}\to K^*}_{max}) \tag{16}$$

which is expected to be valid in the region $q^{2\,\bar{B}\to\rho}_{max} = (m_{\bar{B}} - m_\rho)^2$ and $q^{2\,\bar{B}\to K^*}_{max} = (m_{\bar{B}} - m_{K^*})^2$, respectively. Then the ratio $|V_{ub}|^2/|V_{tb}V^*_{ts}|^2$ is extracted as

$$\begin{aligned}\frac{|V_{ub}|^2}{|V_{tb}V^*_{ts}|^2} &= \frac{q^{2\,\bar{B}\to K^*}_{max}}{q^{2\,\bar{B}\to\rho}_{max}}(\frac{p_{K^*}}{p_\rho})_{lim}(\frac{\alpha_{QED}}{4\pi})^2 2(C_V^2 + C_A^2) \\ &\cdot[\frac{d\Gamma(\bar{B}\to\rho l\bar{\nu})}{dq^2}]_{q^2\to q^{2\,\bar{B}\to\rho}_{max}}/[\frac{d\Gamma(\bar{B}\to K^* l\bar{l})}{dq^2}]_{q^2\to q^{2\,\bar{B}\to K^*}_{max}}.\end{aligned} \tag{17}$$

Here $(p_\rho/p_{K^*})_{lim} = \sqrt{m_\rho/m_{K^*}}$. In the limit $\boldsymbol{p}_{\rho,K^*} \to 0$, i.e., $q^2 \to q^2_{max}$, the q^2 distributions vanish due to the phase space suppression. However, the numerical values of the coefficients of $\boldsymbol{p}_{\rho,K^*}$ in eqs. (8) and (15) can be precisely extracted in experiments. In fact, CLEO collaboration has accurately determined the value of $|V_{cb}|f(q^2_{max})$ for the process $\bar{B} \to D^* l\bar{\nu}$ [8]. In the similar manner, the right-hand side of eq. (17) can be determined by experiments. Expression (17) is our main result. In this expression, V^*_{ts} itself is not directly measured, however it is well determined by the unitarity condition, so $|V_{ub}|$ can be precisely evaluated. Also once $|V_{ub}|$ is known by some other methods, $|V^*_{ts}|$ can be determined from eq. (17). Let us discuss the number of events needed for an analysis of this type. From the figure shown in Lim *et. al.* [5], we can read off

$$\int_{.6}^{.8} ds \frac{dBR.(B\to e^+e^- + anything)}{ds} \sim (3-5)\times 10^{-7}, \tag{18}$$

where $s = q^2/m_b^2$. For a luminosity such that 3×10^7 $B\bar{B}$ pairs are produced, there should be $72 - 120$ $B \to e^+e^-$ *or* $\mu^+\mu^-$ + *anything* events in the kinematic region $.6 < s < .8$ in one year of running the B factory. In this kinematic region, *anything* should be dominated by K^*. If the higher end of the estimate is valid, barring unexpected background or systematic problems, we hope to perform a 10% level measurement of $|V_{ub}|$. The theoretical uncertainty in the derivation of eq. (17) lies in eq. (16), stemming

from the breaking of the $SU(3)$-flavor symmetry in the ρ and K^* mesons. This is expected to be small. For example, we may guess that the ratio of the wave functions of ρ and K^* mesons is estimated by the ratio of their decay constants[9]:

$$\frac{g_{K^*K\pi}}{g_{\rho\pi\pi}} = 1.08 \pm .02. \tag{19}$$

The difference of the Fermi motion of the b quark in the decays $\bar{B} \to \rho l \bar{\nu}$ and $\bar{B} \to K^* l \bar{l}$ may give rise to the error of order $(m_s^2 - m_u^2)/\{(M_{\bar{B}} - m_b)m_b\}$ [10]. An accurate computation of the ratio $f^{K^*}(q^{2\,\bar{B}\to K^*}_{max})/f^{\rho}(q^{2\,\bar{B}\to\rho}_{max})$ will allow a more precise determination of $|V_{ub}|$. Corrections to the relations (13) and (14) are of order Λ_{QCD}/m_b in the zero recoil K^* limit. The uncertainty due to these corrections are significantly reduced in the level of the q^2-distribution (15), because the contributions of the operator O_7 is numerically less than 10 % of those of O_8 and O_9, and accordingly we expect this error to be of order $\Lambda_{QCD}/m_b \times 10\% \sim .4\%$ and is negligible. In general, $C_{V,A}$ in eq. (17) may be sensitive to the parameters of new physics beyond the standard model. This fact provides us with an interesting possibility that a value of $|V_{ub}|$ extracted in our strategy will play a role in probing for new physics, by comparing values of $|V_{ub}|$ determined by other methods. We have used the q^2-distributions of $\bar{B} \to \rho l \bar{\nu}$ and $\bar{B} \to K^* l \bar{l}$ to determine $|V_{ub}|$. Studies on the forward-backward asymmetry of the leptons [11] and the polarization of ρ and K^* mesons may be also useful. The forward-backward asymmetry is described by the form factor $f^{\rho} g^{\rho}$ in the decay $\bar{B} \to \rho l \bar{\nu}$, and is described by $(f^{K^*})^2$, $(g^{K^*})^2$ and $f^{K^*} g^{K^*}$ for $\bar{B} \to K^* l \bar{l}$. The terms $(f^{K^*})^2$ and $(g^{K^*})^2$ come from the magnetic moment type operator O_7, when its hadronic matrix elements are related to those of O_8 and O_9 in the static heavy quark approximation. Along the similar line, an analysis similar to ours can be made using the radiative decay $\bar{B} \to K^* \gamma$ [12]. However the theoretical prediction of this decay rate suffers from the uncertainty due to the large recoil momentum of the K^* meson and the long distance contributions [13]. One of us (A. Y) would like to thank K-I. Izawa, N. Kitazawa, T. Morozumi, M. Tanabashi and S. Uno for useful discussions, and L. T. Handokoo for sending his computer program of the QCD corrected Wilson coefficients.

References

[1] M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. **D23**, 1567 (1981); I. I. Bigi and A. I. Sanda, Phys. Rev. **D29**, 1393 (1984); Nucl. Phys. **B193**, 85 (1981); Comments Nucl. Part. Phys. 14, 149 (1985); Nucl. Phys. **B281**, 41 (1987); I. Dunietz and J. Rosner, Phys. Rev. **D34**, 1404 (1986); D. Du, I. Dunietz and D.Wu, Phys. Rev. **D34**, 3414 (1986).

[3] There investigated several methods for the determinations of $|V_{ub}|$. Inclusive processes are studied in, P. Ball, V. M. Braun and H. G. Dosch, Phys. Rev. **D48**, 2110 (1993); C. S. Kim, P. Ko, D. Hwang and W. Namgung, Phys. Rev. **D50**, 5762 (1994); B. Blok and T. Mannel, Phys. Rev. **D51**, 2208 (1995); C. S. Kim and A. D. Martin, Phys. Lett. **B345**, 296 (1995). Semileptonic decays are used in, G. Kramer, T. Mannel and G. A. Shuler, Z. Phys. **C51**, 649 (1991); D. Du and C. Liu, Phys. Rev. **D50**, 4558 (1994); H. Li and H. L. Yu, Phys. Rev. Lett. **72**, 4388 (1995); N. Kitazawa, Phys. Lett. **B349**, 541 (1995). This analysis is based on the effective Lagrangian obtained in N. Kitazawa and T. Kurimoto, Phys. Lett. **B323**, 65 (1994); A. Datta, UH-511-825-95, hep-ph-9504429; M. Oda, M. Ishida and S. Ishida, NUP-A-94-7. Non-leptonic decays are analyzed in, I. Dunietz and J. Rosner, CERN-TH-5899-90; D. Choudhury, D. Indumati, A. Soni and S. U. Sankar, Phys. Rev. **D45**, 217 (1992); N. G. Deshpande and C. O. Dib, Phys. Lett. **B319**, 313 (1993). The use of the radiative B decays has not been seriously investigated in these analyses.

[4] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981).

[5] C. S. Lim, T. Morozumi and A. I. Sanda, Phys. Lett. **B218**, 343 (1989); N. G. Deshpande, J. Trampetic and K. Panose, Phys. Rev. **D39**, 1461 (1989); P. J. O'Donnell and H. K. K. Tung, Phys. Rev. **D43**, R2067 (1991).

[6] B. Grinstein, M. J. Savage and M. B. Wise, Nucl. Phys. **B319**, 271 (1989); M. Misiak, Nucl. Phys. **B393**, 23 (1993); M. Ciuchini, E. Franco, G. Martinelli

and L. Reina, Nucl. Phys. **B415**, 403 (1994). Misiak denotes our O_8 and O_9 as $\tilde{O}_9$ and $\tilde{O}_{10}$, respectively. Strategies to determine the Wilson coefficients from experiments of rare B decays are discussed e.g., in C. Greub, A. Ioannissian and D. Wyler, Phys. Lett. **B346**, 149 (1995); A. Ali, G. F. Guidice and T. Mannel, CERN-TH7346, hep-ph-9408213.

[7] N. Isgur and M. B. Wise, Phys. Rev. **D42**, 2388 (1990).

[8] CLEO collaboration, Phys. Rev. **D47**, 791 (1993); Phys. Rev. **D51**, 1014 (1994). Their form factor $A_1(q^2)$ is equal to our $(m_{\bar{B}} + m_{D^*})f^{D^*}(q^2)$.

[9] Review of Particle Properties, Phys. Rev. **D50**, 1173 (1994); See also M. Hayakawa, T. Kurimoto and A. I. Sanda, Prog. Theor. Phys. **92**, 377 (1994). It is interesting that the similar estimate is obtained by Burdman and Donoghue in a different context; G. Burdman and J. F. Donoghue, Phys. Lett. **B270**, 55 (1991). Using the Bauer-Stech-Wirbel model, they estimate that the ratio $f^{K^*}(0)/f^{\rho}(0)$ is 1.04×1.10.

[10] I. I. Bigi, M. A. Shifman, N. G. Uraltsev and A. L. Vainstein, Int. J. Modern. Phys. Lett. **A9**, 2467 (1994).

[11] A. Ali, T. Mannel and T. Morozumi study the forward-backward asymmetry in the inclusive level $b \to sl\bar{l}$; A. Ali, T. Mannel and T. Morozumi, Phys. Lett. **B273**, 505 (1991).

[12] Burdman and Donoghue relate the decays $\bar{B} \to K^*\gamma$ and $\bar{B} \to \rho l\bar{\nu}$ at a particular point in the Dalitz plot; Burdman and Donoghue, in Ref. [9]. See also P. J. O'Donnell and H. K. K. Tung, Phys. Rev. **D48**, 2145 (1993).

[13] E. Golowich and S. Pakvasa, Phys. Lett. **B205**, 393 (1988); N. G. Deshpande, J. Trampetic and K. Panose, Phys. Lett. **B214**, 467 (1988).